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Chapter-13

Limits and Derivatives

* $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

* $\lim_{x \to a} [f(x) \times g(x)] = \lim [f(x)] \times \lim_{x \to a} [g(x)]$

* $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

* $\lim_{x \to a} \frac{x^n - a^n}{n - a} = n a^{n-1}$

* $\lim_{x \to 0} \frac{\sin x}{x} = 1$

* $\lim_{x \to 0} = \frac{\tan x}{x} = 1$

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Exercise -13.1

Q Evaluate - - - - - - - - - 22

1.) $\lim_{x\to 3} = x+3$

$\lim_{x\to 3} = 3+3 = 6$

2. $\lim_{x\to 4}\left(x-\frac{22}{7}\right)$

$\lim_{x\to \pi}\left(\pi-\frac{22}{7}\right) \Rightarrow \boxed{\pi-\frac{22}{7}}$

3. $\lim_{r\to 1}\pi r^2$

$\lim_{r\to 1} = \pi(1)^2 = \pi$

4. $\lim_{x\to 4}\frac{4x+3}{x-2}$

$\lim_{x\to 4} = \frac{4(4)+3}{4-2}$

$= \frac{16+3}{2} = \frac{19}{2}$

5. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

$\lim_{x\to -1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1}$

$= \frac{\cancel{1}-\cancel{1}+1}{-2} = \frac{-1}{2}$

6. $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Let $x+1$ be y

$\lim_{y\to 1} = \frac{y^5-1}{y-1}$ $y\to 1$ as $x\to 0$

$= 5\cdot 1^{5-1}$

$\lim_{x\to 0} = 5$ (Ans)

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7. $\lim_{x\to 2} \dfrac{3x^2-x-10}{x^2-4}$

$\Rightarrow \lim_{x\to 2} = \dfrac{3x^2-(6-5)x-10}{(x-2)(x+2)}$

$\Rightarrow \lim_{x\to 2} = \dfrac{3x^2-6x+5x-10}{(x-2)(x+2)}$

$\Rightarrow \lim_{x\to 2} = \dfrac{3x(x-2)+5(x-2)}{(x-2)(x+2)}$

$\Rightarrow \lim_{x\to 2} = \dfrac{(3x+5)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$\Rightarrow \lim_{x\to 2} = \dfrac{3(2)+5}{2+2}$

$\Rightarrow \lim_{x\to 2} = \dfrac{6+5}{4} \Rightarrow \dfrac{11}{4}$

8. $\lim_{x\to 3} \dfrac{x^4-81}{2x^2-5x-3}$

$\Rightarrow \lim_{x\to 3} = \dfrac{(x^2+9)(x^2-9)}{2x^2-(6-1)x-3}$

$\Rightarrow \lim_{x\to 3} = \dfrac{(x^2+9)(x^2-9)}{2x^2-6x+x-3}$

$\Rightarrow \lim_{x\to 3} = \dfrac{(x^2+9)(x^2-9)}{2x(x-3)+1(x-3)}$

$\Rightarrow \lim_{x\to 3} = \dfrac{(x^2+9)(x+3)\cancel{(x-3)}}{2x+1\cancel{(x-3)}}$

$\Rightarrow \lim_{x\to 3} = \dfrac{((3)^2+9)(3+3)}{2(3)+1}$

$\Rightarrow \lim_{x\to 3} = \dfrac{(9+9)(6)}{7}$

$\Rightarrow \dfrac{18\times 6}{7} = \dfrac{108}{7}$ (Ans)

9. $\lim_{x\to 0} \dfrac{ax+b}{(x+1)}$

$\lim_{x\to 0} = \dfrac{a(0)+b}{(0)+1}$

$\lim_{x\to 0} = \dfrac{0+b}{0+1}$

$\lim_{x\to 0} = b \Rightarrow b$

10. $\lim_{x\to 1} \dfrac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

$\Rightarrow \lim_{x\to 1} \left[\dfrac{z^{1/3}-1}{z-1} \div \dfrac{z^{1/6}-1}{z-1}\right]$

$\Rightarrow \lim_{x\to 1} \left[\dfrac{z^{1/3}-1}{z-1}\right] \div \lim_{x\to 1} \left[\dfrac{z^{1/6}-1}{z-1}\right]$

$\Rightarrow n a^{n-1} \mid a=1$

$\Rightarrow \left(\dfrac{1}{3}\right)(a)^{1/3-1} \div \dfrac{1}{6}(a)^{1/6-1}$

$\Rightarrow \left(\dfrac{1}{3}\right)(1)^{\frac{-2}{3}} \div \dfrac{1}{6}(1)^{-5/6}$

$\Rightarrow \dfrac{\frac{1}{3}}{\frac{1}{6}} \Rightarrow \dfrac{\cancel{6}^{\,2}}{\cancel{3}} \Rightarrow \lim_{x\to 1} = 2$ (Ans)

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Chapter-13

Limits and Derivatives

Exercise -13.1

13. $\lim_{x \to 0} \frac{\sin ax}{bx}$

$= \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{ax}{bx}$

$= \left(\lim_{ax \to 0} \frac{\sin ax}{ax} \right) \times \frac{a}{b}$

$= 1 \times \frac{a}{b} \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$

$= \frac{a}{b}$ (Ans)

15. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Let $\pi - x = y$

$\frac{1}{\pi} \left(\lim_{y \to 0} \frac{\sin y}{y} \right)$

$= \frac{1}{\pi} \times 1 = \frac{1}{\pi}$ (Ans)

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17. $\lim_{x\to0}\frac{\cos 2x-1}{\cos x-1}$

$=\lim_{x\to0}\frac{\cancel{1}-2\sin^2x\cancel{-1}}{\cancel{1}-2\sin^2 x/2\cancel{-1}}$

$=\lim_{x\to0}\frac{\cancel{-2}(2\sin x/2\cos x/2)^2}{\cancel{-2}\sin^2 x/2}$

$=\lim_{x\to0}\frac{4\cancel{\sin^2 x/2}\cos^2 x/2}{\cancel{\sin^2 x/2}}$

$=4\cos^2 0/2=4\times1$

$=4$ (Ans)

20. $\lim_{x\to0}\frac{\sin ax+bx}{ax+\sin bx}$ $a, b, a+b\neq0$

$\frac{\lim_{x\to0}\frac{\sin ax}{ax}\times ax+bx}{\lim_{x\to0}ax+\frac{\sin bx}{bx}\times bx}$

$=\frac{x\left[\left(\lim_{ax\to0}\frac{\sin ax}{ax}\right)\times a+b\right]}{x\left[\lim_{x\to0}a+\left(\lim_{bx\to0}\frac{\sin bx}{bx}\right)\times b\right]}$

$\frac{1\times a+b}{a+1\times b}=\frac{\cancel{(a+b)}}{\cancel{(a+b)}}$

$=1$ (Ans)

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Q1. $\lim_{x\to 0} (\operatorname{cosec} x - \cot x)$

$= \lim_{x\to 0} \left[\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right]$

$= \lim_{x\to 0} \left[\frac{1-\cos x}{\sin x}\right]$

$= \lim_{x\to 0} \left\{\frac{\cancel{1}-(\cancel{1}-2\sin^2 x/2)}{\sin x}\right.$

$= \lim_{x\to 0} \frac{\cancel{2}\sin^{\cancel{2}} x/2}{\cancel{2}\cancel{\sin x/2}\cos x/2}$

$= \lim_{x\to 0} \tan x/2$

$= \tan 0 = 0$ (Ans)

* Q2. $\lim_{x\to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

Let $x - \frac{\pi}{2} = y \Rightarrow x = \pi/2 + y$

$\lim_{x\to \frac{\pi}{2}} \frac{\tan 2x}{y}$

$\cancel{\lim_{x\to \frac{\pi}{2}}} = \lim_{y\to 0} \frac{\tan 2(\pi/2 + y)}{y}$

$= \lim_{y\to 0} \frac{\tan(\pi + 2y)}{y}$

$= \left[\lim_{2y\to 0} \frac{\tan 2y}{2y}\right] \times 2 \Rightarrow 1\times 2 = 2$ (Ans)

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*23. Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$

L.H.L

$\lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} 2(0-h)+3$

$= \lim_{h\to 0} -2h+3$

$= -2\times 0+3 = 3$

R.H.L.

$\lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} \cancel{2(0+h)+3}$ $3(x+1)$

$= \lim_{h\to 0} \cancel{2h+3}$ $3((0)+3$

$= 3$

L.H.L = R.H.L

So $\lim_{x\to 0} f(x)$ exist

L.H.L

$\lim_{h\to 0}$ $\cancel{f(}3(x+1)$

$3(1)+3$

$3+3$

$=6$

R.H.L

$\lim_{h\to 0}$ $3(x+1)$

$3(1)+3$

$=6$

L.H.L = R.H.L

So $\lim_{x\to 1} f(x)$ exist

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Q9. Let $a_1, a_2, \ldots a_n$ .... function

$f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$

What is .... $\lim_{x \to a} f(x)$

$f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$

$\lim_{x\to a_1} f(x) \quad \lim_{x \to a_1} f(x) = (a_1-a_1)(a_1-a_2)\ldots(a_1-a_n)$

$= 0\,(a_1-a_2)\ldots(a_1-a_n)$

$= 0$

$\boxed{\lim_{x\to a_1} f(x) = 0}$ Ans

$\lim_{x\to a} f(x) = \lim_{x\to a}(x-a_1)(x-a_2)\ldots(x-a_n)$

$= (a-a_1)(a-a_2)\ldots(a-a_n)$

30. If $f(x) = \begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>0 \end{cases}$ $\quad |x| = \begin{cases} -x & x<0 \\ x & x>0 \end{cases}$

For what value (s) of a does $\lim_{x\to 0} f(x)$ exists?

Sol:- $f(x) = |x|+1, \quad x<0$

$= -x \quad x<0$

Case I

$a<0$

$\cancel{f(x) = -x}$

$\cancel{\lim f(x) =}$

L.H.L.

$\lim_{h\to 0} f(a-h)$

$= \lim_{h\to 0} f(a-h)+1$

$= -(a-h)+1$

$= -(a-0)+1$

$= -a+1$

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R.H.L.

$\lim_{h \to 0} f(a+h)$

$= \lim_{h \to 0} |a+h|+1$

$= \lim_{h \to 0} -(a+h)+1$

$= -(a+0)+1$

$= -a+1$

Case II $a>0$

L.H.L.

$\lim_{h \to 0} f(a-h)$

$= \lim_{h \to 0} |a-h|-1$

$= (a-h)-1$

$= \cancel{-(a-0)-1}$

$= \cancel{-}a-1$

$= (a-1)$

R.H.L.

$\lim_{h \to 0} f(a-h)$

$= \lim_{h \to 0} |a-h|+1$

$= -(a-h)+1$

$= -(a-0)+1$

$= \cancel{-a+1}$

$= (a-1)$

31: $\lim_{x \to 1} \frac{f(x)-2}{x^2-1} = \pi$

$\frac{\lim_{x \to 1} f(x)-2}{\lim_{x \to 1} x^2-1} = \pi$

$\Rightarrow \lim_{x \to 1} f(x)-2 = \pi\left(\lim_{x \to 1} x^2-1\right)$

$= \lim_{x \to 1} f(x)-2 = \pi \times 0$

$= \lim_{x \to 1} f(x)-2 = 0$

$= \lim_{x \to 1} f(x) = 2$ (Ans)

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Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$f(x) = x^n$$

$$f(x+h) = (x+h)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{{}^nC_0 x^n h^0 + {}^nC_1 x^{n-1} h^1 + {}^nC_2 x^{n-2} h^2 + \cdots - x^n}{h}$$

$$= \lim_{h \to 0} \frac{x^n + {}^nC_1 x^{n-1} h + {}^nC_2 x^{n-2} h^2 + \cdots {}^nC_n x^0 h^n - x^n}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}\left[{}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} h + \cdots + {}^nC_n h^{n-1}\right]}{\cancel{h}}$$

$$nx^{n-1} + 0 + 0 + \cdots + 0$$

$$\boxed{f'(x) = nx^{n-1}}$$

$$\boxed{\frac{d}{dx} x^n = nx^{n-1}}$$

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$$\frac{d}{dx}\tan x = \sec^2 x$$

$f(x) = \tan x$

$f(x+h) = \tan(x+h)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right] = \cancel{\lim_{h\to 0} \frac{1}{h}\left[\sin(x+h\right.}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{\cos(x+h)\cos x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin\{x+h-x\}}{\cos(x+h)\cdot\cos x}\right]$$

$$= \left(\lim_{h\to 0} \frac{\sin h}{h}\left(\frac{1}{\lim_{h\to 0}\cos(x+h)\cos x}\right)\right)$$

$$1 \times \frac{1}{\cos(x+0)\cdot\cos x} = \frac{1}{\cos x \cos x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$f'(x) = \tan x = \sec^2 x$

$$\frac{d}{dx} = \tan x = \sec^2 x$$

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Ques - Find the derivative of x^3-27 using first principle

$$f(x) = x^3 - 27$$
$$f(x+h) = (x+h)^3 - 27$$

$$f'x = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^3 - \cancel{27}] - [x^3 - \cancel{27}]}{h} \quad \cancel{x^3+h^3+3hx-27-x^3+27}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+h^3+3xh(x+h)-\cancel{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}[h^2+3x(x+h)]}{\cancel{h}}$$

$$= \lim_{h\to 0} = 0^2 + 3x(x+0)$$
$$= 3x^2 \text{ (Ans)}$$

$$\frac{d}{dx}(x^3-27) = 3x^2 \ \boxed{\text{Ans}}$$

Ques - Find the derivative of $\frac{1}{x^2}$ using first principle

$$f(x) = \frac{1}{x^2}$$

$$f(x+h) = \frac{1}{(x+h)^2}$$

$$f'x = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

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$$f'x = \lim_{h\to 0} \frac{\left(\frac{1}{(x+h)^2}\right) - \frac{1}{x^2}}{h}$$

$$= \cancel{\lim_{h\to 0} \frac{(x+h)^{-2} - (x^{-2})}{h}}$$

$$= \lim_{h\to 0} \frac{\frac{x^2 - (x+h)^2}{x^2 (x+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{x^2 - x^2 - h^2 - 2xh}{h(x^2 (x+h)^2}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-h-2x)}{\cancel{h}(x^2 (x+h)^2}$$

$$= \lim_{h\to 0} \frac{-2\cancel{x}}{x^2 \cdot x^{\cancel{2}}}$$

$$= \lim_{h\to 0} \frac{-2}{x^2} \quad \text{(Ans)}$$

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Q- Find the derivative of $f(x)$ $\sin x$ by first ~~derivative~~ principle

$$f(x) = x\sin x$$
$$f(x+h) = (x+h)\sin(x+h)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x\sin(x+h) + h\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x[\sin(x+h) - \sin x] + h\sin(x+h)}{h}$$

$$= \lim_{h\to 0} \frac{x \times 2\cos\frac{x+h+x}{2} \cdot \sin\frac{x+h-x}{2}}{h} + \lim_{h\to 0} \frac{h\sin(x+h)}{h}$$

$$= \lim_{h\to 0} x \times 2\cos\left(\frac{2x+h}{2}\right)\left(\lim_{\frac{h}{2}\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2} \times 2}\right) + \sin(x+0)$$

$$= x \times \cos\left(\frac{2x+0}{2}\right) \times 1 + \sin x$$

$$= x\cos x + \sin x$$

$$\boxed{\frac{d}{dx}(x\sin x) = x\cos x + \sin x}$$

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Find the derivative of $f(x) = \sin 2x$ by first principle

$$f(x) = \sin 2x$$
$$f(x+h) = \sin 2(x+h)$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin 2(x+h) - \sin 2x}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left[\frac{2(x+h)+2x}{2}\right] \cdot \sin\left[\frac{2(x+h)-2x}{2}\right]}{h}$$

$$= \lim_{h \to 0} 2\cos\left[\frac{\cancel{2}(x+h+x)}{\cancel{2}}\right] \cdot \sin\left[\frac{\cancel{2}(x+h\cancel{-x})}{\cancel{2}}\right]$$

$$= \lim_{h \to 0} \frac{2\cos(2x+h) \cdot \sin h}{h} \quad \cancel{\lim_{h \to 0}}$$

$$= \lim_{h \to 0} 2\cos(2x+h)\left[\frac{\sin h}{h}\right]$$

$$= 2\cos(2x)\,1$$
$$= 2\cos 2x$$

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Derivatives formula:-

$$\frac{d}{dx}\left[u(x)+v(x)\right] = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{du}{dx} - u\frac{du}{dx}}{v^2}$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}\log_e x = \frac{1}{x}$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\log_a x = \frac{1}{x}\log_e a$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = \operatorname{cosec} x \cot x$$

$$\frac{d}{dx}a^x = a^x \log_e a$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

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Exercise-13.2

1\. Find the derivative of x^2-2 at $x=10$.

$$f(x) = x^2-2$$
$$f'(x) = \frac{d}{dx}(x^2) - \frac{d}{dx}(2)$$
$$= 2x - 0$$
$$f'(x) = 2x$$
$$f'(10) = 2\times 10 = 20 \text{ (Ans)}$$

2\. Find the derivative of x at $x=1$

$$y = 99x$$
$$\frac{dy}{dx} = \frac{99\,d(x)}{dx}$$
$$= 99\times 1$$
$$\frac{dy}{dx} = 99$$
$$\left(\frac{dy}{dx}\right)_{x=100} = 99$$

5\. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \cdots + \frac{x^3}{2} + x + 1$$

$$f'(x) = \frac{\cancel{100}x^{99}}{\cancel{100}} + \frac{\cancel{99}x^{98}}{\cancel{99}} \cdots \frac{2x}{2} + 1 + 0$$

$$f'(x) = x^{99} + x^{98} + \cdots + x + 1 \text{ (Ans)}$$
$$f'(0) = 0 + 0 + \cdots + 0 + 1$$
$$\boxed{f'(0) = 1} - 1$$

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$f'(1) = 1^{99} + 1^{98} + \cdots + 1^1 + 1$

$= 1 + 1 + \cdots + 1 + 1$

$f'(1) = 100$

$f'(1) = 100 \times 1$

$\boxed{f'(1) = 100\, f'(0)}$ from (1)

<u>Hence proved</u>

7.

(i) $(x-a)(x-b)$

$f(x) = (x-a)(x-b)$

$f'(x) = (x-a)\frac{d}{dx}(x-b) + (x-b)\frac{d}{dx}(x-a)$

$= (x-a)(1-0) + (x-b)(1-0)$

$x - a + x - b$

$f'(x) = 2x - a - b$ (Ans)

(iii) $\frac{x-a}{x-b}$

$y = \frac{x-a}{x-b}$

$\frac{dy}{dx} = \frac{(x-b)\frac{d}{dx}(x-a) - (x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$

$= \frac{(x-b)\times 1 - (x-a)\times 1}{(x-b)^2}$

$= \frac{\cancel{x} - b - \cancel{x} + a}{(x-b)^2} = \frac{(a-b)}{(x-b)^2}$ (Ans)

9.

(iii) $x^{-3}(5+3x)$

$y = x^{-3}(5+3x)$

$\frac{dy}{dx} = x^{-3}\frac{d}{dx}(5+3x) + (5+3x)\frac{d}{dx}x^{-3}$

$= x^{-3} \times (0+3\times 1) + (5+3x) \times -3x^{-4}$

$= x^{-3} \times 3 - 15x^{-4} - 9x^{-3}$

$= -6x^{-3} - 15x^{-4}$

$= \frac{-6}{x^{3}} - \frac{15}{x^{4}}$

$f(x) = 5x^{-3} + 3x^{1}x^{-3}$

$f(x) = 5x^{-3} + 3x^{-2}$

$f'(x) = 5(-3)x^{-4} + 3(-2)x^{-3}$

$f'(x) = -15x^{-4} - 6x^{-3}$ (Ans)

(v) $x^{-4}(3-4x^{-5})$

$f(x) = 3x^{-4} - 4x^{-5}.x^{-4}$

$f(x) = 3x^{-4} - 4x^{-9}$

$f'(x) = 3(-4)x^{-5} - 4(-9)x^{-10}$

$f'(x) = -12x^{-5} + -39x^{-10}$

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Miscellaneous Exercise

1. Find - - - - - - - - - Principle:

(iv) $\cos\left(x - \frac{\pi}{8}\right)$

$f(x) = \cos(x - \pi/8)$

$f(x+h) = \cos(x + h - \pi/8)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x+h - \pi/8) - \cos(x - \pi/8)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x+h - \cancel{\pi/8}) - x + \cancel{\pi/8}}{h} \quad \frac{2\sin x + y}{2}$$

$$= \cancel{\lim_{h \to 0}} \frac{\cos(x+h) - x}{h} \quad \lim$$

$$= \lim_{h \to 0} \quad \cos$$

$$= \lim_{h \to 0} \frac{-2\sin\left[\frac{x+h-\pi/8+x-\pi/8}{2}\right] \cdot \sin\left[\frac{x+h-\pi/8-x+\pi/8}{2}\right]}{h}$$

$$= \lim_{h \to 0} -\cancel{2}\sin\left[x - \pi/8 + \frac{h}{2}\right] \times \left[\lim_{h \to 0} \frac{\sin h/2}{h/2}\right] \times \cancel{2}$$

$= -\sin(x - \pi/8 + 0) \times 1$

$= -\sin(x - \pi/8)$ Ans

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3. $(px+q)\left(\frac{r}{x}+s\right)$

$f(x) = (px+q)\left(\frac{r}{x}+s\right)$

$f'(x) = (px+q)\frac{d}{dx}(rx^{-1}+s) + \left(\frac{r}{x}+s\right)\frac{d}{dx}(px+q)$

$= (px+q)\,[r\times -1\,x^{-2}+0] + \left(\frac{r}{x}+s\right)(p\times 1+0)$

$= (px+q)\left(-\frac{r}{x^2}\right) + \left(\frac{r}{x}+s\right)p$

$= px\times -\frac{r}{x^2} + q\times -\frac{r}{x^2} + \frac{r}{x}\times p + sp$

$= -\cancel{\frac{pr}{x}} - \frac{qr}{x^2} + \cancel{\frac{rp}{x}} + sp$

$\boxed{= -\frac{qr}{x^2} + sp}$ (Ans)

7. $y = \frac{1}{ax^2+bx+c}$

$y = (ax^2+bx+c)^{-1}$

$\frac{dy}{dx} = \frac{d(ax^2+bx+c)^{-1}}{dx}$

$= \frac{d(ax^2+bx+c)^{-1}}{d(ax^2+bx+c)} \times \frac{d(ax^2+bx+c)}{dx}$

$= (-1)(ax^2+bx+c)^{-2}(a\times 2x + b\times 1+0]$

$= -\frac{2ax+b}{(ax^2+bx+c)}$ (Ans)

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8. $\dfrac{ax+b}{px^2+qx+r}$

$$y = \frac{ax+b}{px^2+qx+r}$$

$$\frac{dy}{dx} = \frac{(px^2+qx+r)\frac{d}{dx}(ax+b) - (ax+b)\frac{d}{dx}(px^2+qx+r)}{(px^2+qx+r)^2}$$

$$= \frac{(px^2+qx+r)(a\times 1+0) - (ax+b)(p\times 2x+q\times 1+0)}{(px^2+qx+r)^2}$$

$$= \frac{a(px^2+qx+r) - (ax+b)(2px+q)}{(px^2+qx+r)^2}$$

$$= \frac{(apx^2+\cancel{aqx}+ar) - (2apx^2+\cancel{aqx}+2bpx+bq)}{(px^2+qx+r)^2}$$

$$= \frac{-apx^2-2bpx+ar-bq}{(px^2+qx+r)^2} \quad \text{(Ans)}$$

13. $(ax+b)^n(cx+d)^m$

Let $y = (ax+b)^n(cx+d)^m$

$$\frac{dy}{dx} = (ax+b)^n \times \frac{d}{dx}(cx+d)^m + (cx+d)^m\frac{d}{dx}(ax+b)^n$$

$$= (ax+b)^n \times \frac{d(cx+d)^m}{d(cx+d)} \times \frac{d(cx+d)}{dx} + (cx+d)^m\frac{d(ax+b)^n}{d(ax+b)} \times \frac{d(ax+b)}{dx}$$

$$= (ax+b)^n \times m(cx+d)^{m-1}(c\times 1+0) + (cx+d)^m n(ax+b)^{n-1}\times(a\times 1+$$

$$= (ax+b)^n \times m(cx+d)^{m-1}\times c + (cx+d)^m n(ax+b)^{n-1}\cdot a$$

$$= (ax+b)^{n-1}(cx+d)^{m-1}[cm(ax+b)+an(cx+d)] \quad \text{Ans}$$

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17. $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$

$$\frac{dy}{dx} = \frac{(\sin x - \cos x)\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2}$$

$$= -\frac{[\sin^2 x + \cos^2 x - \cancel{2\sin x\cos x} + \sin^2 x + \cos^2 x + \cancel{2\sin x\cos x}]}{(\sin x - \cos x)^2}$$

$$= \frac{-(1+1)}{(\sin x - \cos x)^2} = \frac{-2}{(\sin x - \cos x)^2} \text{ (Ans)}$$

18. $\dfrac{a + b\sin x}{c + d\cos x}$

$y = \dfrac{a + b\sin x}{c + d\cos x}$

$$\frac{dy}{dx} = \frac{(c + d\cos x)\frac{d}{dx}(a + b\sin x) - (a + b\sin x)\frac{d}{dx}(c + d\cos x)}{(c + d\cos x)^2}$$

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$$\frac{dy}{dx} = \cancel{(c+d\cos x)}$$

$$= \frac{(c+d\cos x)\, b\cos x - (a+b\sin x)\,(d\sin x)}{(c+d\cos x)^2}$$

$$= \frac{bc\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c+d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd(\cos^2 x + \sin^2 x)}{(c+d\cos x)^2}$$

$$f'(x) = \frac{bc\cos x + ad\sin x + bd}{(c+d\cos x)^2} \quad \text{(Ans)}$$

21. $f(x)\ \dfrac{\sin(x+a)}{\cos x}$

$$f'(x) = \frac{\cos x \dfrac{d}{dx}(\sin(x+a)) - \sin(x+a)\dfrac{d}{dx}(\cancel{\sin}\cos x)}{\cos^2 x}$$

$$\cancel{= \cos\cos} = \frac{\cos x\cos(x+a) + \sin(x+a)\sin x}{\cos^2 x}$$

$$f'(x) = \frac{\cos(x+a-x)}{\cos^2 x}$$

$$f'(x) = \frac{\cos a}{\cos^2 x} \quad \text{(Ans)}$$

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21. $\frac{\sin(x+a)}{\cos x}$

$y = \frac{\sin(x+a)}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx} \sin(x+a) - \sin(x+a) \frac{d}{dx} \cos x}{\cos^2 x}$

$= \frac{\cos x \cos(x+a) - \sin(x+a)(-\sin x)}{\cos^2 x}$

$= \frac{\cos(x+a)\cos x + \sin(x+a)\sin x}{\cos^2 x}$

$= \frac{\cos[(x+a)-x]}{\cos^2 x}$

$= \frac{\cos a}{\cos^2 x}$ (Ans)

23. $(x^2+1)\cos x$

$y = (x^2+1)\cos x$

$\frac{dy}{dx} = (x^2+1)\frac{d}{dx}\cos x + \cos x \frac{d}{dx}(x^2+1)$

$= (x^2+1)(-\sin x) + \cos x\,(2x+0)$

$= -(x^2+1)\sin x + 2x\cos x$

$= -x^2\sin x - \sin x + 2x\cos x$ (Ans)

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25. $(x+\cos x)(x-\tan x)$

$$\frac{dy}{dx} = (x+\cos x)\frac{d}{dx}(x-\tan x) + (x-\tan x)\frac{d}{dx}(x+\cos x)$$

$$= (x+\cos x)(1-\sec^2 x) + (x-\tan x)(1-\sin x)$$

$$= (x+\cos x)(-\tan^2 x) + (x-\tan x)(1-\sin x) \text{ Ans}$$

$$= -\tan^2 x\,(x+\cos x) + (x-\tan x)(1-\sin x)$$

27. $\dfrac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

$$y = \frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$$

$$\frac{dy}{dx} = \cos \pi/4 \,\frac{d}{dx}\left(\frac{x^2}{\sin x}\right)$$

$$= \cos \pi/4 \,\frac{\sin x \frac{d}{dx} x^2 - x^2 \frac{d}{dx}\sin x}{\sin^2 x}$$

$$= \cos \pi/4 \,\frac{2x\sin x - x^2\cos x}{\sin^2 x} \quad (\text{Ans})$$

29. $(x+\sec x)(x-\tan x)$

$y=(x+\sec x)(x-\tan x)$

$$\frac{dy}{dx}=(x+\sec x)\frac{d}{dx}(x-\tan x)+(x-\tan x)\frac{d}{dx}(x+\sec x)$$

$$=(x+\sec x)\cancel{\frac{d}{dx}}(1-\sec^2 x)+(x-\tan x)\overset{(1+\sec x\tan x)}{\cancel{\frac{d}{dx}(x+\sec x)}}$$

Ans

30. $\dfrac{x}{\sin^n x}$

$y=\dfrac{x}{\sin^n x}$

$$\frac{dy}{dx}=\frac{x}{\sin^n x}$$

$$\frac{dy}{dx}=\cancel{x\frac{d}{dx}}\ \frac{\sin^n x\frac{d}{dx}(x)-x\frac{d}{dx}\sin^n x}{(\sin^n x)^2}$$

$$=\cancel{\sin^n x\frac{d}{dx}(x)-x\frac{d}{dx}s}$$

$$=\frac{\sin^n x\times 1-x\times n\sin^{n-1}x\times\cos x}{\sin^{2n}x}$$

$$=\frac{\sin^{n-1}x\,[\sin x-nx\cos x]}{\sin^{2n}x}$$

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$$= \frac{[\sin x - nx \cos x]}{\sin^{2n-n+1} x}$$

$$= \frac{\sin x - nx \cos x}{\sin^{n+1} x} \quad \text{(Ans)}$$

Ques - $y = \frac{a + b\sin x}{c + d\cos x}$

$$\frac{dy}{dx} = \frac{c + d\cos x \frac{d}{dx}(a + b\sin x) - a + b\sin x \frac{d}{dx}(c + d\sin x)}{\cancel{a+b}\,(c + d\cos x)^2}$$

$$= \frac{(c + d\cos x)\, b\cos x - (a + b\sin x)(d\sin x)}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd(\cos^2 x + \sin^2 x)}{(c + d\cos x)^2}$$

$$f'(x) = \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2} \quad \text{(Ans)}$$